Large Scale Graph Processing - GraphX, Giraph++, and Pegasus

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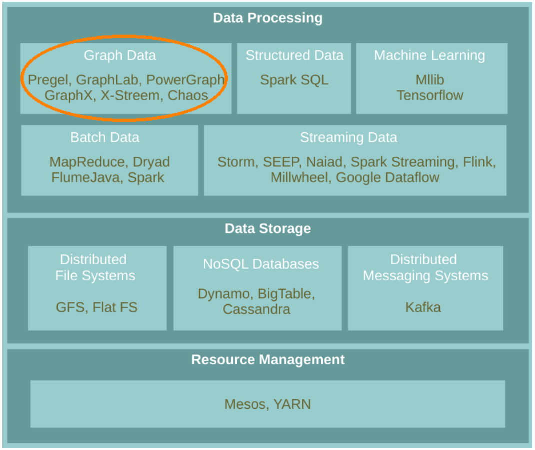
The Course Web Page

`https://id2221kth.github.io`

Where Are We?



Data Processing
Graph Data
Pregel, GraphLab, PowerGraph
GraphX, X-Streem, Chaos
Structured Data
Spark SQL
Machine Learning
Mllib
Tensorflow
Batch Data
MapReduce, Dryad
FlumeJava, Spark
Streaming Data
Storm, SEEP, Naiad, Spark Streaming, Flink,
Millwheel, Google Dataflow
Data Storage
Distributed
File Systems
GFS, Flat FS
NoSQL Databases
Dynamo, BigTable,
Cassandra
Distributed
Messaging Systems
Kafka
Resource Management
Mesos, YARN

Graph Algorithms Challenges

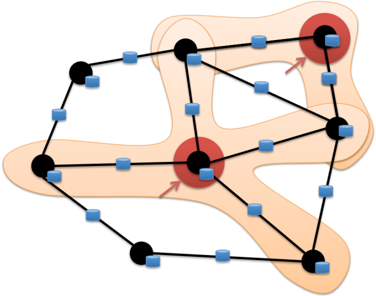
- Difficult to extract parallelism based on partitioning of the data.
- Difficult to express parallelism based on partitioning of computation.

Different Approached to Process Large Scale Graphs

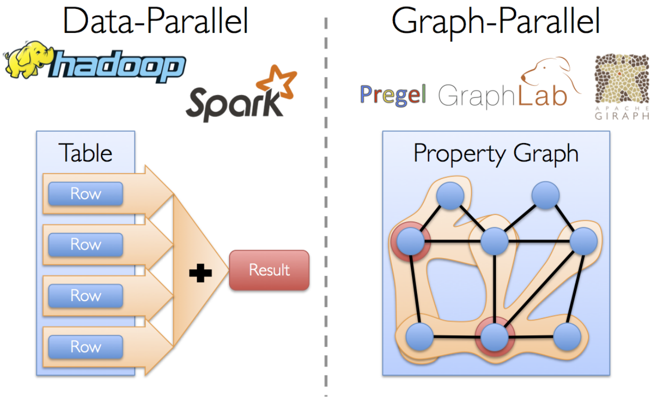
- Think like a vertex
- Think like an edge
- Think like a table
- Think like a graph
- Think like a matrix

Think Like a Table

Graph-Parallel Processing Model



Data-Parallel vs. Graph-Parallel Computation



Motivation (2/3)

- Graph-parallel computation: restricting the types of computation to achieve performance.

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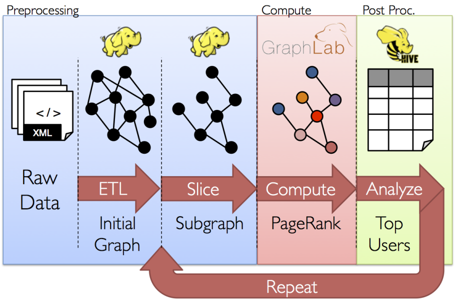
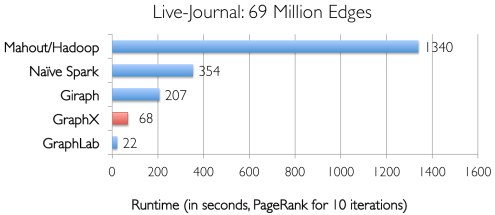
- Graph-parallel computation: restricting the types of computation to achieve performance.
- The same restrictions make it difficult and inefficient to express many stages in a typical graph-analytics pipeline.



Motivation (3/3)



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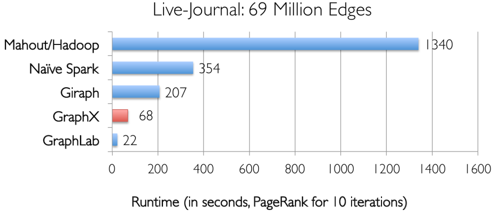
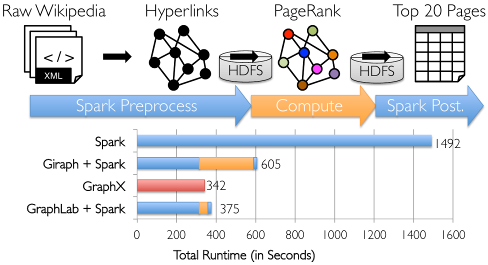
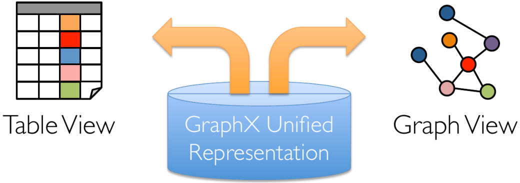




Think Like a Table

- Unifies data-parallel and graph-parallel systems.
- Tables and Graphs are composable views of the same physical data.



GraphX

GraphX

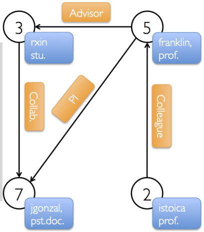
- GraphX is the library to perform graph-parallel processing in Spark.
- In-memory caching.
- Lineage-based fault tolerance.

The Property Graph Data Model

- Spark represent graph structured data as a property graph.
- It is logically represented as a pair of vertex and edge property collections.
  - `VertexRDD` and `EdgeRDD`

```scala
// VD: the type of the vertex attribute
// ED: the type of the edge attribute
class Graph[VD, ED] {
  val vertices: VertexRDD[VD]
  val edges: EdgeRDD[ED]
}
```

Property Graph



Vertex Table

Id	Property (V)
3	(rxin, student)
7	(jgonzal, postdoc)
5	(franklin, professor)
2	(istoica, professor)

Edge Table

SrcId	DstId	Property (E)
3	7	Collaborator
5	3	Advisor
2	5	Colleague
5	7	PI

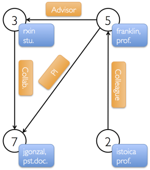
The Vertex Collection

- `VertexRDD`: contains the vertex properties keyed by the vertex ID.

```scala
class Graph[VD, ED] {
  val vertices: VertexRDD[VD]
  val edges: EdgeRDD[ED]
}

// VD: the type of the vertex attribute
abstract class VertexRDD[VD] extends RDD[(VertexId, VD)]
```

Property Graph



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2	(istoica, professor)

Edge Table

SrcId	DstId	Property (E)
3	7	Collaborator
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Vertices:

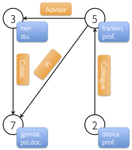
The Edge Collection

- EdgeRDD: contains the edge properties keyed by the source and destination vertex IDs.

```scala
class Graph[VD, ED] {
  val vertices: VertexRDD[VD]
  val edges: EdgeRDD[ED]
}

// ED: the type of the edge attribute
case class Edge[ED](srcId: VertexId, dstId: VertexId, attr: ED)
abstract class EdgeRDD[ED] extends RDD[Edge[ED]]
```

Property Graph



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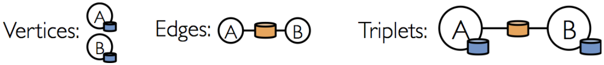
Edges: 

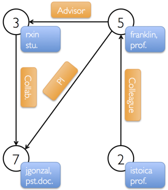
The Triplet Collection

- The triplets collection consists of each edge and its corresponding source and destination vertex properties.
- It logically joins the vertex and edge properties: `RDD[EdgeTriplet[VD, ED]]`.
- The `EdgeTriplet` class extends the `Edge` class by adding the `srcAttr` and `dstAttr` members, which contain the source and destination properties respectively.



Building a Property Graph

Property Graph



Vertex Table

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Edge Table

SrcId	DstId	Property (E)
3	7	Collaborator
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```scala
val users: RDD[(VertexId, (String, String))] = sc.parallelize(Array((3L, ("rxin", "student")),
  (7L, ("jgonzal", "postdoc")), (5L, ("franklin", "prof")), (2L, ("istoica", "prof"))))

val relationships: RDD[Edge[String]] = sc.parallelize(Array(Edge(3L, 7L, "collab"),
  Edge(5L, 3L, "advisor"), Edge(2L, 5L, "colleague"), Edge(5L, 7L, "pi"), Edge(5L, 1L, "-")))

val defaultUser = ("John Doe", "Missing")

val graph: Graph[(String, String), String] = Graph(users, relationships, defaultUser)
```

Graph Operators

- Information about the graph
- Property operators
- Structural operators
- Joins
- Aggregation
- Iterative computation
- ...

Information About The Graph (1/2)

- Information about the graph

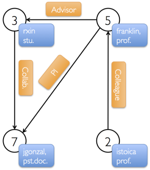
```
val numEdges: Long
val numVertices: Long
val inDegrees: VertexRDD[Int]
val outDegrees: VertexRDD[Int]
val degrees: VertexRDD[Int]
```

- Views of the graph as collections

```
val vertices: VertexRDD[VD]
val edges: EdgeRDD[ED]
val triplets: RDD[EdgeTriplet[VD, ED]]
```

Information About The Graph (2/2)

Property Graph



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SrcId	DstId	Property (E)
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```scala
// Constructed from above
val graph: Graph[(String, String), String]

// Count all users which are postdocs
graph.vertices.filter { case (id, (name, pos)) => pos == "postdoc" }.count

// Count all the edges where src > dst
graph.edges.filter(e => e.srcId > e.dstId).count
```

Property Operators

- Transform vertex and edge attributes
- Each of these operators yields a new graph with the vertex or edge properties modified by the user defined `map` function.

```
def mapVertices[VD2](map: (VertexId, VD) => VD2): Graph[VD2, ED]
def mapEdges[ED2](map: Edge[ED] => ED2): Graph[VD, ED2]
def mapTriplets[ED2](map: EdgeTriplet[VD, ED] => ED2): Graph[VD, ED2]
```

Property Operators

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```
def mapVertices[VD2](map: (VertexId, VD) => VD2): Graph[VD2, ED]
def mapEdges[ED2](map: Edge[ED] => ED2): Graph[VD, ED2]
def mapTriplets[ED2](map: EdgeTriplet[VD, ED] => ED2): Graph[VD, ED2]
```

```
val relations: RDD[String] = graph.triplets.map(triplet =>
    triplet.srcAttr._1 + " is the " + triplet.attr + " of " + triplet.dstAttr._1)
relations.collect.foreach(println)
```

Property Operators

- Transform vertex and edge attributes
- Each of these operators yields a new graph with the vertex or edge properties modified by the user defined `map` function.

```
def mapVertices[VD2](map: (VertexId, VD) => VD2): Graph[VD2, ED]
def mapEdges[ED2](map: Edge[ED] => ED2): Graph[VD, ED2]
def mapTriplets[ED2](map: EdgeTriplet[VD, ED] => ED2): Graph[VD, ED2]
```

```
val relations: RDD[String] = graph.triplets.map(triplet =>
    triplet.srcAttr._1 + " is the " + triplet.attr + " of " + triplet.dstAttr._1)
relations.collect.foreach(println)
```

```
val newGraph = graph.mapTriplets(triplet =>
    triplet.srcAttr._1 + " is the " + triplet.attr + " of " + triplet.dstAttr._1)
newGraph.edges.collect.foreach(println)
```

Structural Operators

- `reverse` returns a new graph with all the edge directions reversed.

```
def reverse: Graph[VD, ED]
def subgraph(epred: EdgeTriplet[VD, ED] => Boolean, vpred: (VertexId, VD) => Boolean):
    Graph[VD, ED]
def mask[VD2, ED2](other: Graph[VD2, ED2]): Graph[VD, ED]
```

Structural Operators

- `reverse` returns a new graph with all the edge directions reversed.
- `subgraph` takes vertex/edge predicates and returns the graph containing only the vertices/edges that satisfy the given predicate.

```
def reverse: Graph[VD, ED]
def subgraph(epred: EdgeTriplet[VD, ED] => Boolean, vpred: (VertexId, VD) => Boolean):
    Graph[VD, ED]
def mask[VD2, ED2](other: Graph[VD2, ED2]): Graph[VD, ED]
```

```
// Remove missing vertices as well as the edges to connected to them
val validGraph = graph.subgraph(vpred = (id, attr) => attr._2 != "Missing")
graph.vertices.collect.foreach(println)
validGraph.vertices.collect.foreach(println)

// Restrict the answer to the valid subgraph
val validUserGraph = graph.mask(validGraph)
```

Structural Operators

- `reverse` returns a new graph with all the edge directions reversed.
- `subgraph` takes vertex/edge predicates and returns the graph containing only the vertices/edges that satisfy the given predicate.
- `mask` constructs a subgraph of the input graph.

```
def reverse: Graph[VD, ED]
def subgraph(epred: EdgeTriplet[VD, ED] => Boolean, vpred: (VertexId, VD) => Boolean):
    Graph[VD, ED]
def mask[VD2, ED2](other: Graph[VD2, ED2]): Graph[VD, ED]
```

```
// Remove missing vertices as well as the edges to connected to them
val validGraph = graph.subgraph(vpred = (id, attr) => attr._2 != "Missing")
graph.vertices.collect.foreach(println)
validGraph.vertices.collect.foreach(println)

// Restrict the answer to the valid subgraph
val validUserGraph = graph.mask(validGraph)
```

Join Operators

- `joinVertices` joins the vertices with the input RDD.
  - Returns a new graph with the vertex properties obtained by applying the user defined `map` function to the result of the joined vertices.
  - Vertices without a matching value in the RDD retain their original value.

```
def joinVertices[U](table: RDD[(VertexId, U)])(map: (VertexId, VD, U) => VD): Graph[VD, ED]
```

Join Operators

- `joinVertices` joins the vertices with the input RDD.
  - Returns a new graph with the vertex properties obtained by applying the user defined `map` function to the result of the joined vertices.
  - Vertices without a matching value in the RDD retain their original value.

```
def joinVertices[U](table: RDD[(VertexId, U)])(map: (VertexId, VD, U) => VD): Graph[VD, ED]
```

```
val rdd: RDD[(VertexId, String)] = sc.parallelize(Array((3L, "phd")))
val joinedGraph = graph.joinVertices(rdd)((id, user, role) => (user._1, role + " " + user._2))
joinedGraph.vertices.collect.foreach(println)
```

Aggregation (1/2)

- aggregateMessages applies a user defined sendMsg function to each edge triplet in the graph and then uses the mergeMsg function to aggregate those messages at their destination vertex.

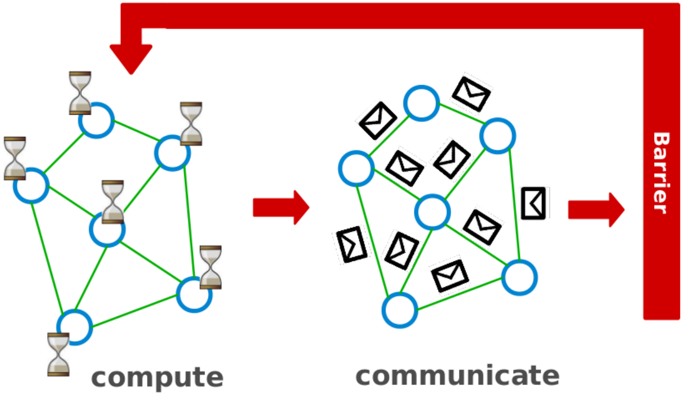
```
def aggregateMessages[Msg: ClassTag](
  sendMsg: EdgeContext[VD, ED, Msg] => Unit, // map
  mergeMsg: (Msg, Msg) => Msg, // reduce
  tripletFields: TripletFields = TripletFields.All):
  VertexRDD[Msg]
```

Aggregation (2/2)

```
// count and list the name of friends of each user
val profs: VertexRDD[(Int, String)] = validUserGraph.aggregateMessages[(Int, String)](
  // map
  triplet => {
    triplet.sendToDst((1, triplet.srcAttr._1))
  },
  // reduce
  (a, b) => (a._1 + b._1, a._2 + " " + b._2)
)

profs.collect.foreach(println)
```

Iterative Computation (1/9)

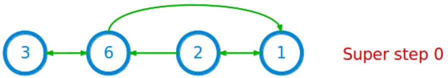


Iterative Computation (2/9)

```
i_val := val

for each message m
  if m > val then val := m

if i_val == val then
  vote_to_halt
else
  for each neighbor v
    send_message(v, val)
```

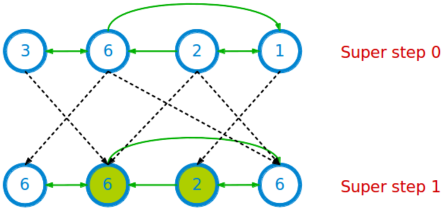


Iterative Computation (3/9)

```
i_val := val

for each message m
  if m > val then val := m

if i_val == val then
  vote_to_halt
else
  for each neighbor v
    send_message(v, val)
```

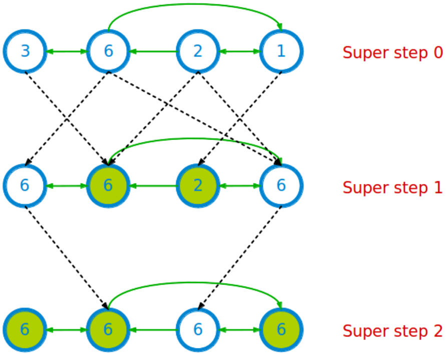


Iterative Computation (4/9)

```
i_val := val

for each message m
  if m > val then val := m

if i_val == val then
  vote_to_halt
else
  for each neighbor v
    send_message(v, val)
```

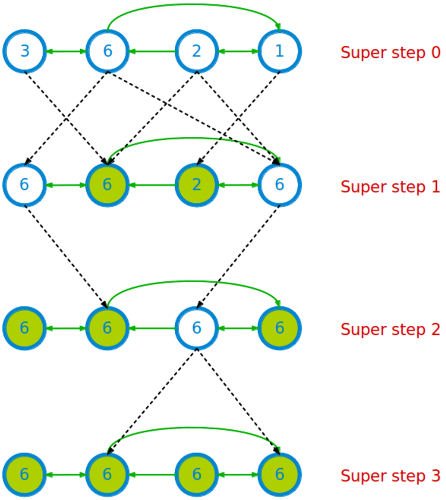


Iterative Computation (5/9)

```
i_val := val

for each message m
  if m > val then val := m

if i_val == val then
  vote_to_halt
else
  for each neighbor v
    send_message(v, val)
```



Iterative Computation (6/9)

- `pregel` takes two argument lists: `graph.pregel(list1)(list2)`.

```
def pregel[A]
  (initialMsg: A, maxIter: Int = Int.MaxValue, activeDir: EdgeDirection = EdgeDirection.Out)
  (vprog: (VertexId, VD, A) => VD, sendMsg: EdgeTriplet[VD, ED] => Iterator[(VertexId, A)],
    mergeMsg: (A, A) => A):
  Graph[VD, ED]
```

Iterative Computation (6/9)

- `pregel` takes two argument lists: `graph.pregel(list1)(list2)`.
- The first list contains configuration parameters
  - The initial message, the maximum number of iterations, and the edge direction in which to send messages.

```
def pregel[A]
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```
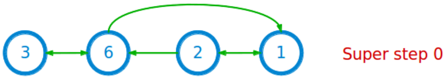

Iterative Computation (6/9)

- `pregel` takes two argument lists: `graph.pregel(list1)(list2)`.
- The first list contains configuration parameters
  - The initial message, the maximum number of iterations, and the edge direction in which to send messages.
- The second list contains the user defined functions.
  - Gather: `mergeMsg`, Apply: `vprog`, Scatter: `sendMsg`

```scala
def pregel[A]
  (initialMsg: A, maxIter: Int = Int.MaxValue, activeDir: EdgeDirection = EdgeDirection.Out)
  (vprog: (VertexId, VD, A) => VD, sendMsg: EdgeTriplet[VD, ED] => Iterator[(VertexId, A)],
    mergeMsg: (A, A) => A):
  Graph[VD, ED]
```

Iterative Computation (7/9)

```
import org.apache.spark._
import org.apache.spark.graphx._
import org.apache.spark.rdd.RDD

val initialMsg = -9999

val vertices: RDD[(VertexId, (Int, Int))] =  sc.parallelize(Array((1L, (1, -1)),
  (2L, (2, -1)), (3L, (3, -1)), (6L, (6, -1))))

val relationships: RDD[Edge[Boolean]] = sc.parallelize(Array(Edge(1L, 2L, true),
  Edge(2L, 1L, true), Edge(2L, 6L, true), Edge(3L, 6L, true), Edge(6L, 1L, true),
  Edge(6L, 3L, true)))

val graph = Graph(vertices, relationships)
```



Iterative Computation (8/9)

```
// Gather: the function for combining messages
def mergeMsg(msg1: Int, msg2: Int): Int = math.max(msg1, msg2)
```

Iterative Computation (8/9)

```scala
// Gather: the function for combining messages
def mergeMsg(msg1: Int, msg2: Int): Int = math.max(msg1, msg2)
```

```scala
// Apply: the function for receiving messages
def vprog(vertexId: VertexId, value: (Int, Int), message: Int): (Int, Int) = {
  if (message == initialMsg)
    value
  else
    (math.max(message, value._1), value._1)
}
```

Iterative Computation (8/9)

```
// Gather: the function for combining messages
def mergeMsg(msg1: Int, msg2: Int): Int = math.max(msg1, msg2)
```

```
// Apply: the function for receiving messages
def vprog(vertexId: VertexId, value: (Int, Int), message: Int): (Int, Int) = {
  if (message == initialMsg)
    value
  else
    (math.max(message, value._1), value._1)
}
```

```
// Scatter: the function for computing messages
def sendMsg(triplet: EdgeTriplet[(Int, Int), Boolean]): Iterator[(VertexId, Int)] = {
  val sourceVertex = triplet.srcAttr
  if (sourceVertex._1 == sourceVertex._2)
    Iterator.empty
  else
    Iterator((triplet.dstId, sourceVertex._1))
}
```

Iterative Computation (9/9)

```
val minGraph = graph.pregel(initialMsg,
                            Int.MaxValue,
                            EdgeDirection.Out)(
                            vprog, // apply
                            sendMsg, // scatter
                            mergeMsg) // gather

minGraph.vertices.collect.foreach{
  case (vertexId, (value, original_value)) => println(value)
}
```

GraphFrames

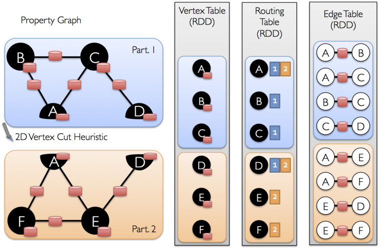
- GraphFrames extends GraphX to provide a `DataFrame` API.
- To build a GraphFrame we need to define the vertices and edges as `DataFrames`.
- `spark-shell --packages graphframes:graphframes:0.6.0-spark2.3-s_2.11`
  - You may need to delete `.ivy2` from your home folder.

Querying the GraphFrames

```
import org.graphframes._
import org.apache.spark.sql.SQLContext

val sqlContext = new org.apache.spark.sql.SQLContext(sc)

val userDF = sqlContext.createDataFrame(Array(("rxin", "student"), ("jgonzal", "postdoc"),
  ("franklin", "prof"), ("istoica", "prof"))).toDF("id", "role")

val relationshipsDF = sqlContext.createDataFrame(Array(("rxin", "jgonzal", "collab"),
  ("franklin", "rxin", "advisor"), ("istoica", "franklin", "colleague"),
  ("franklin", "franklin", "pi"))).toDF("src", "dst", "relationship")

val graphDF = GraphFrame(userDF, relationshipsDF)

graphDF.edges.where("src = 'franklin'").groupBy("src", "dst").count().show
```

Graph Representation

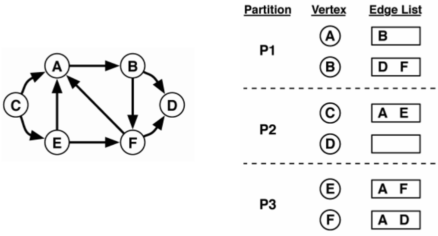
- Vertex-cut partitioning
- Representing graphs using two RDDs: edge-collection and vertex-collection
- Routing table: a logical map from a vertex id to the set of edge partitions that contains adjacent edges.



Think Like a Graph

Motivation (1/2)

- Vertex-centric programming model.
  - Operate on a vertex and its edges.
  - Communication to other vertices, via message passing (Pregel), or shared memory (GraphLab).
- Divide input graphs into partitions.



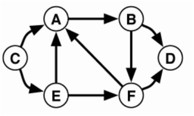
Motivation (2/2)

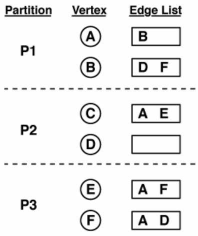
- In the vertex-centric model, a vertex is very short sighted.
  - A vertex has information about its immediate neighbors.
  - Information is propagated through graphs slowly, one hop at a time.

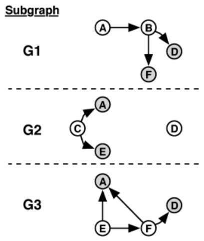
- Graph-centric programming paradigm is proposed to overcome this limitation.

Think Like a Graph

	Think Like a Vertex	Think Like a Graph
Partition	A collection of vertices	A proper subgraph
Computaion	A vertex and its edges	A subgraph
Communication	1-hop at a time, e.g., A $\rightarrow$ B $\rightarrow$ D	Multiple-hops at a time, e.g., A $\rightarrow$ D



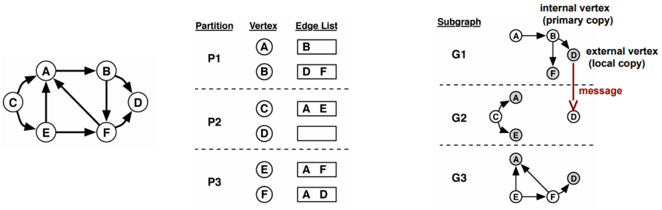




Giraph++

Giraph++

- Expose subgraphs to programmers.
- Internal vertices vs. boundary vertices.



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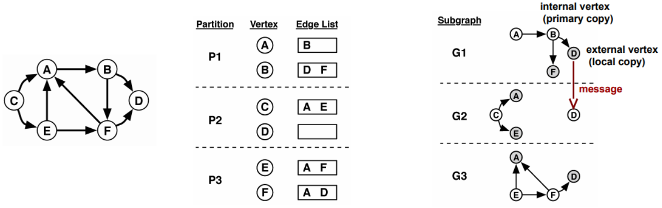
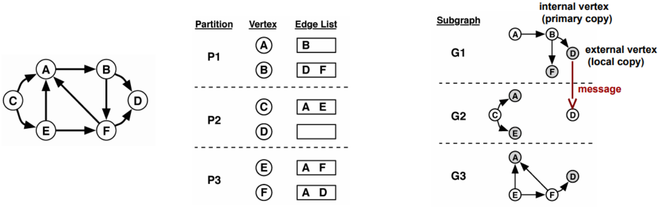
  - Information exchange between internal vertices of a partition is immediate.
  - Messages are only sent from boundary vertices of a partition to internal vertices of a different partition.



Giraph++

- Expose subgraphs to programmers.
- Internal vertices vs. boundary vertices.
  - Information exchange between internal vertices of a partition is immediate.
  - Messages are only sent from boundary vertices of a partition to internal vertices of a different partition.
- A vertex is an internal vertex in exactly one subgraph, but it can be a boundary vertex in zero or more subgraphs.



Execution Model (1/2)

- A program is executed in sequence of supersteps.
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- Supersteps are separated by global synchronization barriers.
- In each superstep, the computation is performed on the whole subgraph in a partition.
- Like in Pregel, each internal vertex of a partition has two states: active or inactive.
- A boundary vertex does not have any state.

Execution Model (2/2)

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- What messages can be used in local computation?
  - External messages from previous superstep (global synchronous computation).
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- This is called hybrid execution model.

Think Like a Matrix

Graphs and Matrices (1/2)

- A graph can be represented by an adjacency matrix.
- Operations on graphs can be performed by algebraic operations on matrices.
- Linear algebra and matrix theory can be applied to solve graph problems.

Graphs and Matrices (2/2)

- Given a graph $\mathtt{G} = (\mathtt{V}, \mathtt{E})$
- Adjacency matrix $\mathtt{A}(\mathtt{G})$, a $|\mathtt{V}| \times |\mathtt{V}|$ matrix

$$\mathtt{A[i][j]} = \begin{cases} 1 & \mathtt{if}\ \mathtt{i} \neq \mathtt{j}\ \mathtt{and}\ (\mathtt{v_i}, \mathtt{v_j}) \in \mathtt{E} \\ 0 & \mathtt{if}\ \mathtt{i} \neq \mathtt{j}\ \mathtt{and}\ (\mathtt{v_i}, \mathtt{v_j}) \notin \mathtt{E} \\ 0 & \mathtt{if}\ \mathtt{i} = \mathtt{j} \end{cases}$$

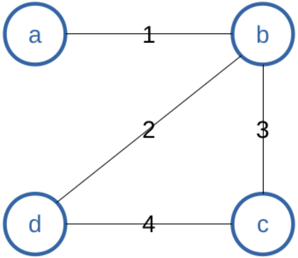
Adjacency Matrix Example

- Produce a vector representing the neighbors of a vertex $\mathtt{v_i}$.

Adjacency Matrix Example

- Produce a vector representing the neighbors of a vertex $\mathtt{v_i}$.
- By computing $\mathbf{A} \cdot \mathbf{x}_{\mathtt{v_i}}$
  - $\mathbf{x}_{\mathtt{v_i}}[\mathtt{i}] = \mathtt{1}$ and all other elements of $\mathbf{x}_{\mathtt{v_i}}$ are 0.
- For example, to find the neighbors of vertex $\mathtt{b}$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = [1\ 0\ 1\ 1]$$



Pegasus

Generalized Iterated Matrix-Vector (GIM-V)

- Targets at iterative graph algorithms.
- Generalized Iterated Matrix-Vector multiplication (GIM-V)
  - Matrix-vector multiplication
  - Assume $\mathbf{M}$ is a $\mathtt{n} \times \mathtt{n}$ matrix, $\mathbf{v}$ is a vector of size $\mathtt{n}$, and $\mathtt{m}_{\mathtt{i},\mathtt{j}}$ denotes the $(\mathtt{i}, \mathtt{j})$ element of $\mathbf{M}$.
  - $\mathbf{v} \leftarrow \mathbf{M} \cdot \mathbf{v}$, where $\mathtt{v}_\mathtt{i} \leftarrow \sum_{\mathtt{j}=1}^{\mathtt{n}} \mathtt{m}_{\mathtt{i},\mathtt{j}} \mathtt{v}_\mathtt{j}$.

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  - $\mathbf{v} \leftarrow \mathbf{M} \cdot \mathbf{v}$, where $v_i \leftarrow \sum_{j=1}^{n} m_{i,j} v_j$.
- Pegasus models each iteration of the graph computation by a GIM-V operation
  - It is repeated until the vertex values in the vector converge.

GIM-V Operators

- $\mathbf{v} \leftarrow \mathbf{M} \cdot \mathbf{v}$, where $\mathtt{v_i} \leftarrow \sum_{\mathtt{j}=1}^{\mathtt{n}} \mathtt{m_{i,j} v_j}$.

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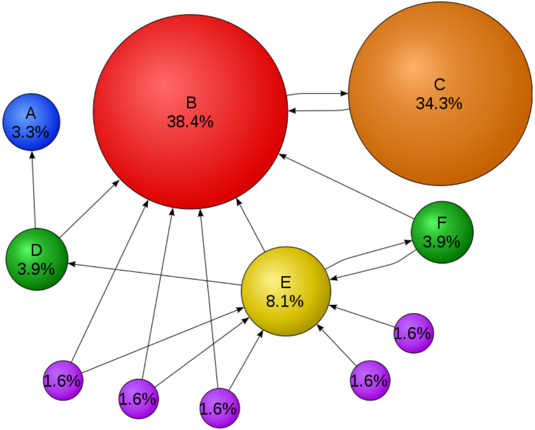
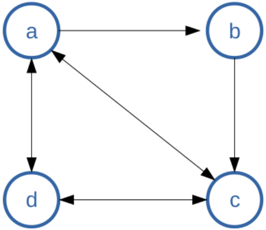
- `combineAll(i)`: for each v_i, to combine all the n intermediate results produced by `combine2` into a single value.
- `assign`: to overwrite the old value of v_i with the new value.

GIM-V Example: PageRank (1/3)



GIM-V Example: PageRank (2/3)



- PageRank formula: $\mathbf{v} \leftarrow (0.85\mathbf{E}^{\mathrm{T}} + 0.15\mathbf{U}) \cdot \mathbf{v}$.
  - $\mathbf{v}$ is a column vector with $\mathtt{n}$ elements.
  - $\mathbf{E}$ is a is the row-normalized adjacency matrix.
  - $\mathbf{U}$ is a $\mathtt{n} \times \mathtt{n}$ matrix, with all elements set to $\frac{1}{\mathtt{n}}$.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 1/2 & 0 \end{bmatrix} \quad \mathbf{U} = \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{bmatrix} \quad \mathbf{v}_{\mathtt{init}} = \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix}$$

- If $\mathtt{M} = 0.85\mathbf{E}^{\mathrm{T}} + 0.15\mathbf{U}$, then we can write the PageRank as $\mathbf{v} \leftarrow \mathbf{M} \cdot \mathbf{v}$.

GIM-V Example: PageRank (3/3)

- PageRank formula: $\mathbf{v} \leftarrow (0.85\mathbf{E}^T + 0.15\mathbf{U}) \cdot \mathbf{v}$.
- $\mathtt{combine2}(\mathtt{i}, \mathtt{j}) = 0.85 \times \mathtt{m}_{\mathtt{i},\mathtt{j}} \times \mathtt{v}_{\mathtt{j}}$
- $\mathtt{combineAll}(\mathtt{i}) = \frac{0.15}{\mathtt{n}} + \sum_{\mathtt{j}=1}^{\mathtt{n}} \mathtt{combine2}(\mathtt{i}, \mathtt{j})$
- $\mathtt{assign}$: $\mathtt{v}_{\mathtt{i}} \leftarrow \mathtt{combineAll}(\mathtt{i})$

Summary

Summary

- Think like a table
  - Graphx: unifies data-parallel and graph-parallel systems.
- Think like a graph
  - Giraph++: exposes subgraphs to programmers
- Think like a matrix
  - Pegasus: linear algebra and matrix theory to solve graph problems.

References

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Questions?