



Large Scale Graph Processing

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The Course Web Page

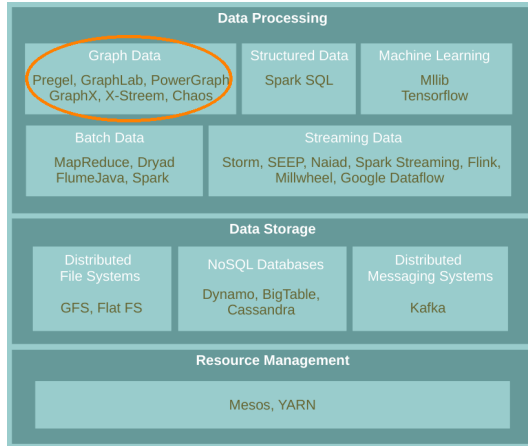
<https://id2221kth.github.io>



The Questions-Answers Page

<https://tinyurl.com/hk7hzpw5>

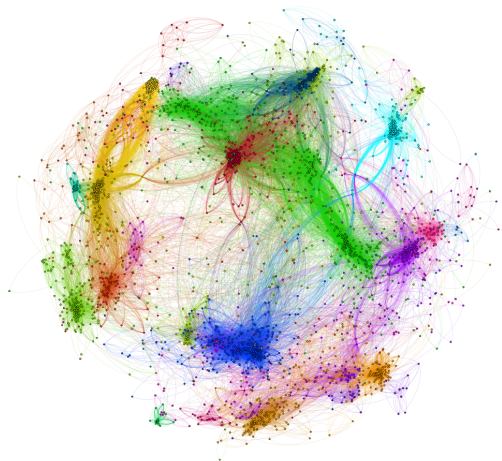
Where Are We?



- ▶ A flexible abstraction for describing relationships between discrete objects.



Large Graph





Graph Algorithms Challenges

- ▶ Difficult to extract **parallelism** based on **partitioning** of **the data**.
- ▶ Difficult to express **parallelism** based on **partitioning** of **computation**.

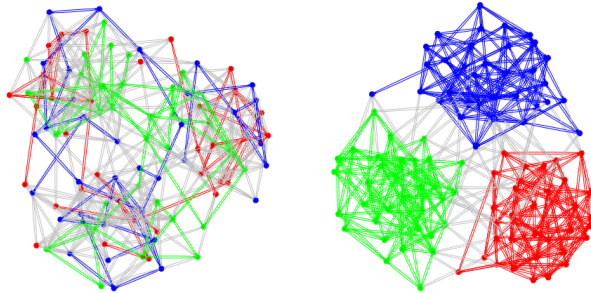


Graph Algorithms Challenges

- ▶ Difficult to extract **parallelism** based on **partitioning** of **the data**.
- ▶ Difficult to express **parallelism** based on **partitioning** of **computation**.
- ▶ **Graph partition** is a challenging problem.

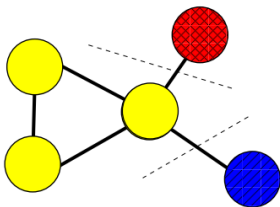
Graph Partitioning

- ▶ Partition large scale graphs and **distribut** to hosts.



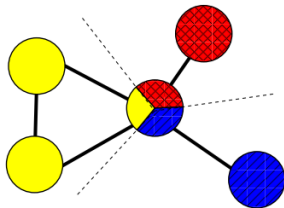
Edge-Cut Graph Partitioning

- ▶ Divide **vertices** of a graph into **disjoint clusters**.
- ▶ Nearly **equal size** (w.r.t. the number of **vertices**).
- ▶ With the **minimum number of edges** that **span separated clusters**.



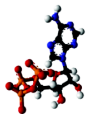
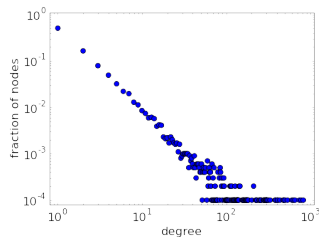
Vertex-Cut Graph Partitioning

- ▶ Divide **edges** of a graph into **disjoint clusters**.
- ▶ Nearly **equal size** (w.r.t. the number of **edges**).
- ▶ With the **minimum** number of **replicated vertices**.

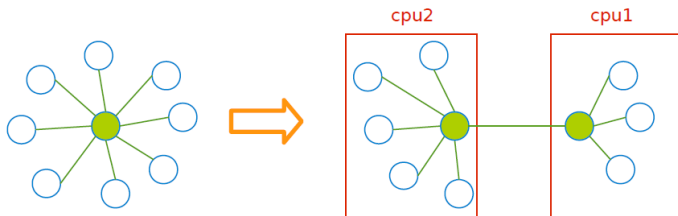
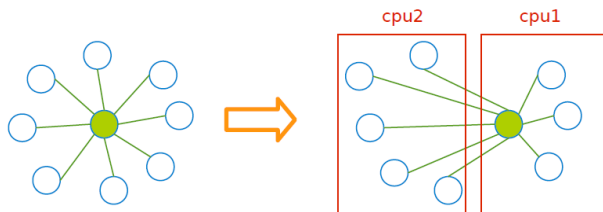


Edge-Cut vs. Vertex-Cut Graph Partitioning (1/2)

- ▶ Natural graphs: skewed **Power-Law** degree distribution.
- ▶ **Edge-cut** algorithms perform **poorly** on Power-Law Graphs.

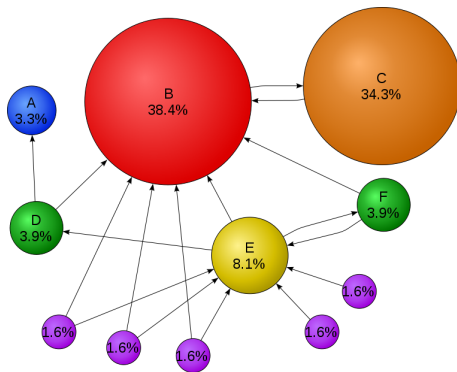


Edge-Cut vs. Vertex-Cut Graph Partitioning (2/2)





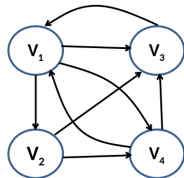
PageRank with MapReduce



$$R[i] = \sum_{j \in \text{Nbrs}(i)} w_{ji} R[j]$$

PageRank Example (1/2)

▶ $R[i] = \sum_{j \in \text{Nbrs}(i)} w_{ji} R[j]$

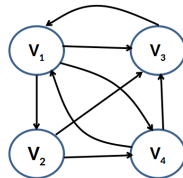


PageRank Example (1/2)

▶
$$R[i] = \sum_{j \in \text{Nbrs}(i)} w_{ji} R[j]$$

▶ Input

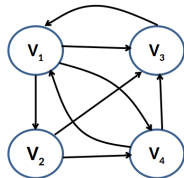
V1: [0.25, V2, V3, V4]
 V2: [0.25, V3, V4]
 V3: [0.25, V1]
 V4: [0.25, V1, V3]



PageRank Example (1/2)

▶
$$R[i] = \sum_{j \in \text{Nbrs}(i)} w_{ji} R[j]$$

▶ Input



V1: [0.25, V2, V3, V4]

V2: [0.25, V3, V4]

V3: [0.25, V1]

V4: [0.25, V1, V3]

▶ Share the rank among all outgoing links

V1: (V2, 0.25/3), (V3, 0.25/3), (V4, 0.25/3)

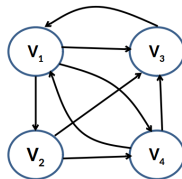
V2: (V3, 0.25/2), (V4, 0.25/2)

V3: (V1, 0.25/1)

V4: (V1, 0.25/2), (V3, 0.25/2)

PageRank Example (2/2)

$$\blacktriangleright R[i] = \sum_{j \in \text{Nbrs}(i)} w_{ji} R[j]$$



V1: (V2, 0.25/3), (V3, 0.25/3), (V4, 0.25/3)

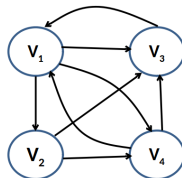
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PageRank Example (2/2)

$$\blacktriangleright R[i] = \sum_{j \in \text{Nbrs}(i)} w_{ji} R[j]$$



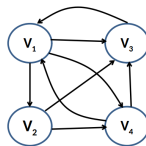
V1: (V2, 0.25/3), (V3, 0.25/3), (V4, 0.25/3)
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 V3: (V1, 0.25/1)
 V4: (V1, 0.25/2), (V3, 0.25/2)

► Output after one iteration

V1: [0.37, V2, V3, V4]
 V2: [0.08, V3, V4]
 V3: [0.33, V1]
 V4: [0.20, V1, V3]

PageRank in MapReduce - Map (1/2)

- ▶ Map function

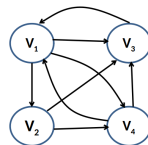


```
map(key: [url, pagerank], value: outlink_list)
  for each outlink in outlink_list:
    emit(key: outlink, value: pagerank / size(outlink_list))

emit(key: url, value: outlink_list)
```

PageRank in MapReduce - Map (1/2)

► Map function



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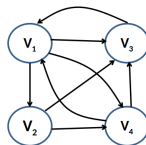
► Input (key, value)

```
((V1, 0.25), [V2, V3, V4])
((V2, 0.25), [V3, V4])
((V3, 0.25), [V1])
((V4, 0.25), [V1, V3])
```



PageRank in MapReduce - Map (2/2)

- ▶ Map function

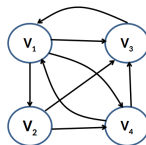


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emit(key: url, value: outlink_list)
```

- ▶ Intermediate (key, value)

```
(V2, 0.25/3), (V3, 0.25/3), (V4, 0.25/3), (V3, 0.25/2), (V4, 0.25/2), (V1, 0.25/1),
(V1, 0.25/2), (V3, 0.25/2)
(V1, [V2, V3, V4])
(V2, [V3, V4])
(V3, [V1])
(V4, [V1, V3])
```




PageRank in MapReduce - Shuffle

► Intermediate (key, value)

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(V2, 0.25/3), (V3, 0.25/3), (V4, 0.25/3), (V3, 0.25/2), (V4, 0.25/2), (V1, 0.25/1),  
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(V2, [V3, V4])  
(V3, [V1])  
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```



PageRank in MapReduce - Shuffle

► Intermediate (key, value)

```
(V2, 0.25/3), (V3, 0.25/3), (V4, 0.25/3), (V3, 0.25/2), (V4, 0.25/2), (V1, 0.25/1),  
(V1, 0.25/2), (V3, 0.25/2)  
(V1, [V2, V3, V4])  
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(V3, [V1])  
(V4, [V1, V3])
```

► After shuffling

```
(V1, 0.25/1), (V1, 0.25/2), (V1, [V2, V3, V4])  
(V2, 0.25/3), (V2, [V3, V4])  
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(V4, 0.25/3), (V4, 0.25/2), (V4, [V1, V3])
```



PageRank in MapReduce - Reduce (1/2)

▶ Reduce function

```
reducer(key: url, value: list_pr_or_urls)
  outlink_list = []
  pagerank = 0

  for each pr_or_urls in list_pr_or_urls:
    if is_list(pr_or_urls):
      outlink_list = pr_or_urls
    else
      pagerank += pr_or_urls

  emit(key: [url, pagerank], value: outlink_list)
```



PageRank in MapReduce - Reduce (1/2)

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▶ Input of the Reduce function

```
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(V2, 0.25/3), (V2, [V3, V4])
(V3, 0.25/3), (V3, 0.25/2), (V3, 0.25/2), (V3, [V1])
(V4, 0.25/3), (V4, 0.25/2), (V4, [V1, V3])
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PageRank in MapReduce - Reduce (2/2)

▶ Reduce function

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```
((V1, 0.37), [V2, V3, V4])
((V2, 0.08), [V3, V4])
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Problems with MapReduce for Graph Analytics

- ▶ MapReduce does **not directly support iterative** algorithms.
 - Invariant graph-topology-data **re-loaded** and **re-processed** at each iteration is **wasting** I/O, network bandwidth, and CPU



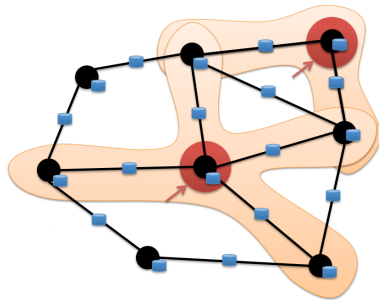
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- ▶ **Materializations** of intermediate results at every MapReduce iteration **harm performance**.

Think Like a Vertex

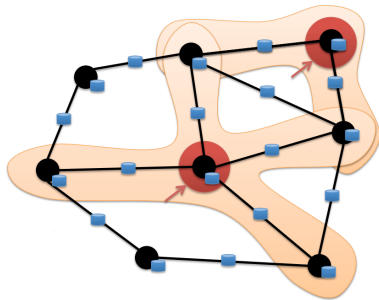
Think Like a Vertex

- ▶ Each vertex computes **individually** its value (in **parallel**).
- ▶ Computation typically depends on the **neighbors**.

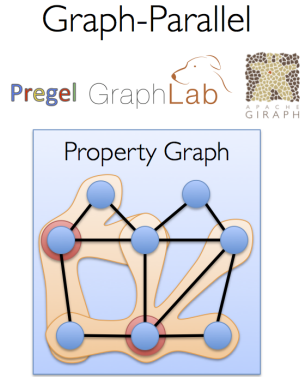
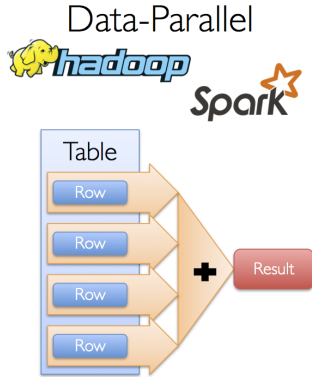


Think Like a Vertex

- ▶ Each vertex computes **individually** its value (in **parallel**).
- ▶ Computation typically depends on the **neighbors**.
- ▶ Also know as **graph-parallel** processing model.

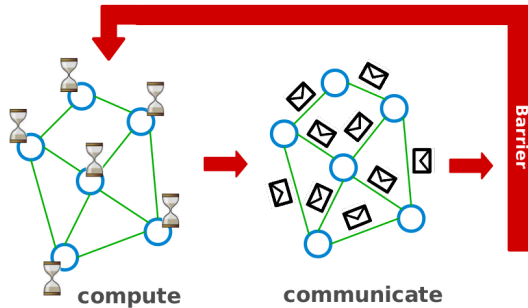


Data-Parallel vs. Graph-Parallel Computation



Pregel

- ▶ Large-scale **graph-parallel** processing platform developed at Google.
- ▶ Inspired by **bulk synchronous parallel (BSP)** model.





Execution Model (1/2)

- ▶ Applications run in sequence of **iterations**, called **supersteps**.



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- ▶ A vertex in superstep **S** can:
 - **reads** messages sent to it in superstep **S-1**.
 - **sends** messages to other vertices: receiving at superstep **S+1**.
 - **modifies** its state.

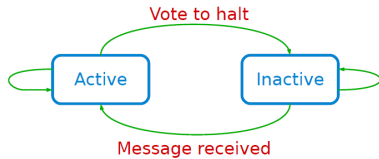


Execution Model (1/2)

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 - **reads** messages sent to it in superstep **S-1**.
 - **sends** messages to other vertices: receiving at superstep **S+1**.
 - **modifies** its state.
- ▶ Vertices communicate directly with one another by **sending messages**.

Execution Model (2/2)

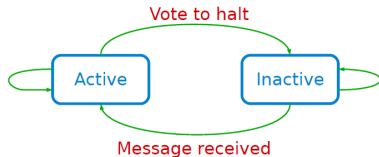
- ▶ Superstep 0: all vertices are in the active state.





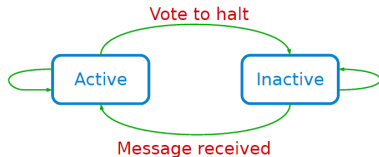
Execution Model (2/2)

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- ▶ A vertex deactivates itself by voting to halt: no further work to do.



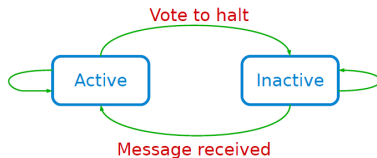
Execution Model (2/2)

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- ▶ A halted vertex can be active if it receives a message.



Execution Model (2/2)

- ▶ Superstep 0: all vertices are in the active state.
- ▶ A vertex deactivates itself by voting to halt: no further work to do.
- ▶ A halted vertex can be active if it receives a message.
- ▶ The whole algorithm terminates when:
 - All vertices are simultaneously inactive.
 - There are no messages in transit.



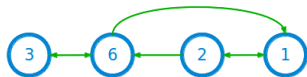


Example: Max Value (1/4)

```
i_val := val

for each message m
  if m > val then val := m

if i_val == val then
  vote_to_halt
else
  for each neighbor v
    send_message(v, val)
```



Super step 0

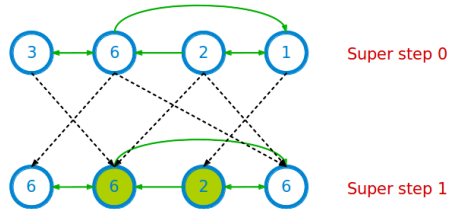
Example: Max Value (2/4)

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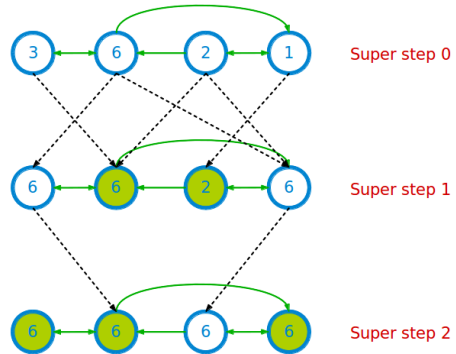
if i_val == val then
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    send_message(v, val)
  
```



Example: Max Value (3/4)

```

i_val := val
for each message m
  if m > val then val := m
if i_val == val then
  vote_to_halt
else
  for each neighbor v
    send_message(v, val)
  
```



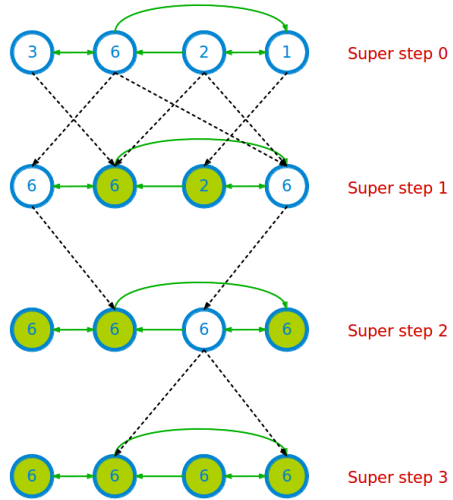
Example: Max Value (4/4)

```

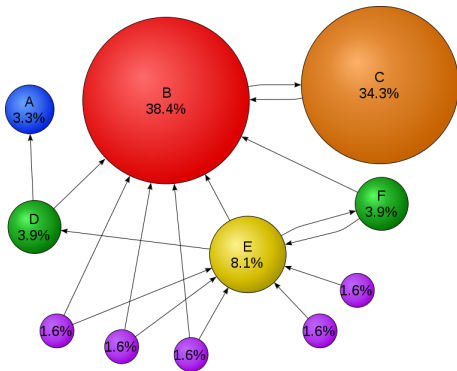
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for each message m
  if m > val then val := m

if i_val == val then
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else
  for each neighbor v
    send_message(v, val)
  
```



Example: PageRank



$$R[i] = \sum_{j \in \text{Nbrs}(i)} w_{ji} R[j]$$



Example: PageRank

```
Pregel_PageRank(i, messages):  
  // receive all the messages  
  total = 0  
  foreach(msg in messages):  
    total = total + msg  
  
  // update the rank of this vertex  
  R[i] = total  
  
  // send new messages to neighbors  
  foreach(j in out_neighbors[i]):  
    sendmsg(R[i] * wij) to vertex j
```

$$R[i] = \sum_{j \in \text{Nbrs}(i)} w_{ji} R[j]$$

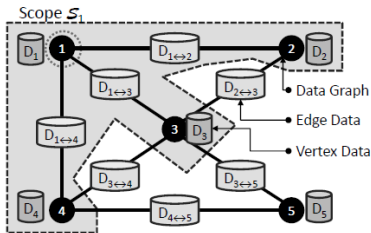
GraphLab/Turi



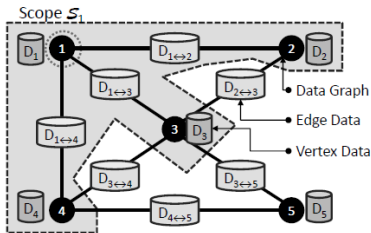
GraphLab

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- ▶ **Vertex scope** of **vertex v** : the data stored in v , and in all **adjacent vertices and edges**.



- ▶ GraphLab allows **asynchronous** iterative computation.
- ▶ **Vertex scope** of **vertex v** : the data stored in v , and in all **adjacent vertices and edges**.
- ▶ A vertex can **read** and **modify** any of the data in its **scope** (**shared memory**).





Example: PageRank (GraphLab)

```
GraphLab_PageRank(i)
  // compute sum over neighbors
  total = 0
  foreach(j in in_neighbors(i)):
    total = total + R[j] * wji

  // update the PageRank
  R[i] = total

  // trigger neighbors to run again
  foreach(j in out_neighbors(i)):
    signal vertex-program on j
```

$$R[i] = \sum_{j \in \text{Nbrs}(i)} w_{ji} R[j]$$



Gather-Apply-Scatter (GAS)

- ▶ **Factorizes** the **local vertices functions** into the **Gather**, **Apply** and **Scatter** phases.
- ▶ **Gather**: **accumulate** information from neighborhood.
- ▶ **Apply**: **apply** the accumulated value to center vertex.
- ▶ **Scatter**: **update** adjacent edges and vertices.



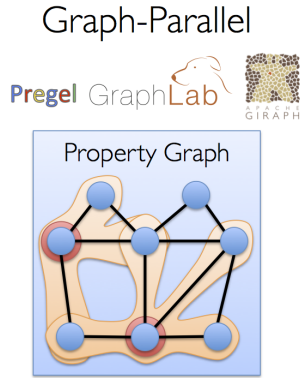
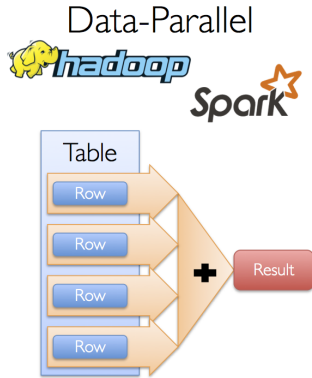
Example: PageRank (GraphLab - GAS)

```
PowerGraph_PageRank(i):  
  Gather(j -> i):  
    return wji * R[j]  
  
  sum(a, b):  
    return a + b  
  
  // total: Gather and sum  
  Apply(i, total):  
    R[i] = total  
  
  Scatter(i -> j):  
    if R[i] changed then activate(j)
```

$$R[i] = \sum_{j \in \text{Nbrs}(i)} w_{ji} R[j]$$

Think Like a Table

Data-Parallel vs. Graph-Parallel Computation



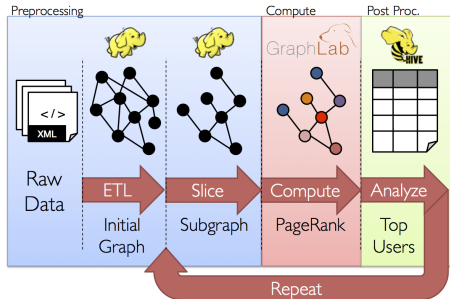


Motivation (2/3)

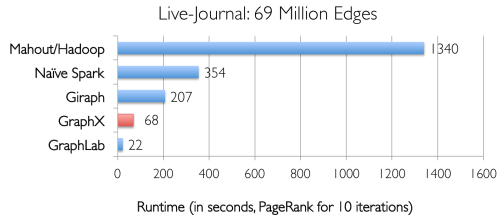
- ▶ **Graph-parallel** computation: **restricting** the types of computation to achieve **performance**.

Motivation (2/3)

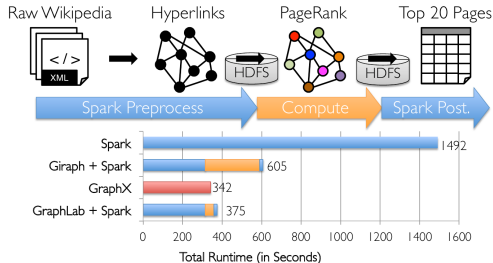
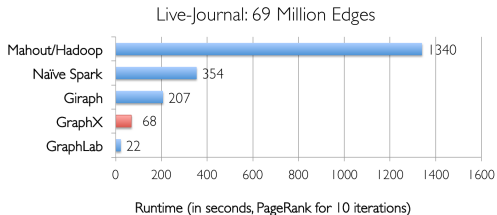
- ▶ **Graph-parallel** computation: **restricting** the types of computation to achieve **performance**.
- ▶ The same restrictions make it **difficult** and **inefficient** to express many stages in a typical graph-analytics **pipeline**.



Motivation (3/3)

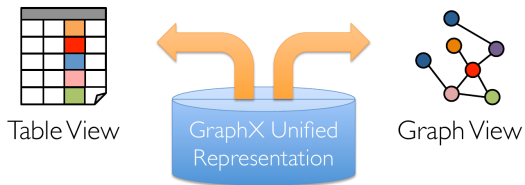


Motivation (3/3)



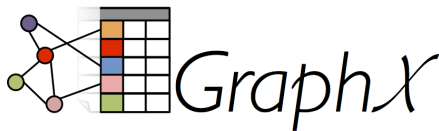
Think Like a Table

- ▶ Unifies **data-parallel** and **graph-parallel** systems.
- ▶ **Tables** and **Graphs** are **composable views** of the **same physical data**.



GraphX

- ▶ **GraphX** is the library to perform **graph-parallel** processing in **Spark**.

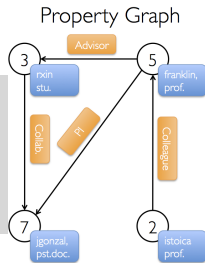


The Property Graph Data Model

- ▶ Spark represent **graph** structured data as a **property graph**.
- ▶ It is logically represented as a pair of **vertex** and **edge property collections**.
 - **VertexRDD** and **EdgeRDD**

```

// VD: the type of the vertex attribute
// ED: the type of the edge attribute
class Graph[VD, ED] {
  val vertices: VertexRDD[VD]
  val edges: EdgeRDD[ED]
}
    
```



Vertex Table

Id	Property (V)
3	(rxin, student)
7	(jgonzal, postdoc)
5	(franklin, professor)
2	(istoica, professor)

Edge Table

SrcId	DstId	Property (E)
3	7	Collaborator
5	3	Advisor
2	5	Colleague
5	7	PI

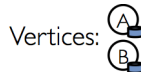
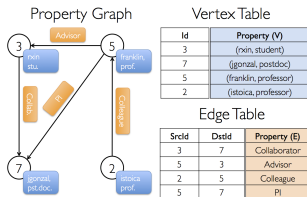
The Vertex Collection

- ▶ **VertexRDD**: contains the vertex properties **keyed by the vertex ID**.

```
class Graph[VD, ED] {
  val vertices: VertexRDD[VD]
  val edges: EdgeRDD[ED]
}
```

// VD: the type of the vertex attribute

```
abstract class VertexRDD[VD] extends RDD[(VertexId, VD)]
```

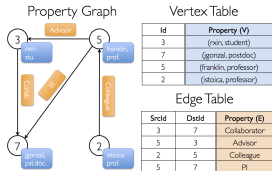


The Edge Collection

- ▶ **EdgeRDD**: contains the edge properties **keyed by the source and destination vertex IDs**.

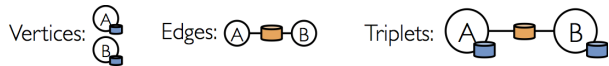
```
class Graph[VD, ED] {
  val vertices: VertexRDD[VD]
  val edges: EdgeRDD[ED]
}

// ED: the type of the edge attribute
case class Edge[ED](srcId: VertexId, dstId: VertexId, attr: ED)
abstract class EdgeRDD[ED] extends RDD[Edge[ED]]
```



The Triplet Collection

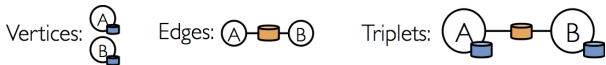
- ▶ The **triplets collection** consists of each **edge** and its **corresponding source and destination vertex** properties.





The Triplet Collection

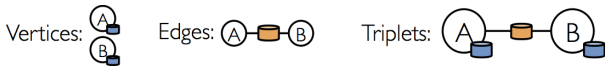
- ▶ The **triplets collection** consists of each **edge** and its **corresponding source and destination vertex** properties.
- ▶ It logically **joins the vertex and edge properties**: `RDD[EdgeTriplet[VD, ED]]`.



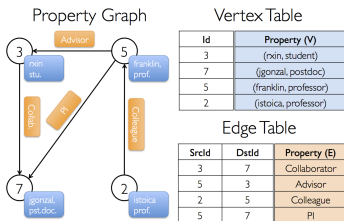


The Triplet Collection

- ▶ The **triplets collection** consists of each **edge** and its **corresponding source and destination vertex** properties.
- ▶ It logically **joins the vertex and edge properties**: `RDD[EdgeTriplet[VD, ED]]`.
- ▶ The `EdgeTriplet` class extends the `Edge` class by adding the `srcAttr` and `dstAttr` members, which contain the **source and destination properties** respectively.

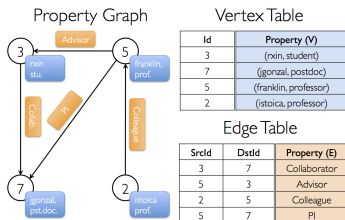


Building a Property Graph



```
val users: RDD[(VertexId, (String, String))] = sc.parallelize(Array((3L, ("rxin", "student")),
    (7L, ("jgonzal", "postdoc")), (5L, ("franklin", "prof")), (2L, ("istoica", "prof"))))
```

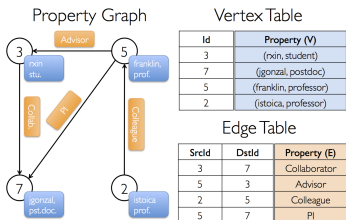
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```

```
val relationships: RDD[Edge[String]] = sc.parallelize(Array(Edge(3L, 7L, "collab"),
    Edge(5L, 3L, "advisor"), Edge(2L, 5L, "colleague"), Edge(5L, 7L, "pi"), Edge(5L, 1L, "-")))
```

Building a Property Graph

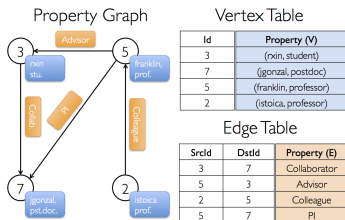


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```

```
val defaultUser = ("John Doe", "Missing")
```

Building a Property Graph



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val users: RDD[(VertexId, (String, String))] = sc.parallelize(Array((3L, ("rxin", "student")),
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```

```
val defaultUser = ("John Doe", "Missing")
```

```
val graph: Graph[(String, String), String] = Graph(users, relationships, defaultUser)
```



Graph Operators

- ▶ Information about the graph
- ▶ Property operators
- ▶ Structural operators
- ▶ Joins
- ▶ Aggregation
- ▶ Iterative computation
- ▶ ...

Summary



Summary

- ▶ Think like a vertex
 - Pregel: BSP, synchronous parallel model, message passing
 - GraphLab: asynchronous model, shared memory, GAS

- ▶ Think like a table
 - Graphx: unifies data-parallel and graph-parallel systems.



References

- ▶ G. Malewicz et al., “Pregel: a system for large-scale graph processing”, ACM SIGMOD 2010
- ▶ Y. Low et al., “Distributed GraphLab: a framework for machine learning and data mining in the cloud”, VLDB 2012
- ▶ J. Gonzalez et al., “Powergraph: distributed graph-parallel computation on natural graphs”, OSDI 2012
- ▶ J. Gonzalez et al., “GraphX: Graph Processing in a Distributed Dataflow Framework”, OSDI 2014

Questions?